21369 Practice Midterm 1

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1 Problem 1

a. Perhaps we are trying to approximate the value of $\sqrt{42}$ with Newton's Method. Choose some polynomial f, with $\sqrt{42}$ as a root, and use two iterations of Newton's Method on f with initial approximation $x_0 = 6$.

b. If $f(x)$ is the polynomial you used in part a, let $g(x) = f(x)^2$ on $\mathbb{R}$. Why would it be unwise to use $g(x)$ instead of $f(x)$ with Newton's Method to approximate $\sqrt{42}$?

2 Problem 2

Let f be a class C^∞ function over $\mathbb{R}$ (that is, f can be differentiated infinitely many times). Consider approximating f'' with an expansion of the form

$$Af(x+h^2) + Bf(x+h) - Bf(x-h) - Af(x-h^2)$$

where h is a "small" deviation and $A, B \in \mathbb{R}$. Demonstrate using Taylor's Theorem that no such approximation exists, at least without adding other terms.

3 Problem 3

Let f be a class C^2 function over $\mathbb{R}$ (that is, f can be differentiated twice and f'' is continuous), where $f''(x) \geq 0$ on $\mathbb{R}$ and r is a simple root of f. A student wonders if we can modify Newton's Method to depend on x_{n-1} and x_n instead of just x_n and get a faster convergence rate to the root as a result. The student devises the **Weighted Newton Method**, which is defined as

$$x_{n+1} = \frac{2}{3}\left(x_n + \frac{f(x_n)}{f'(x_n)}\right) + \frac{1}{3}\left(x_{n-1} + \frac{f(x_{n-1})}{f'(x_{n-1})}\right)$$

for $n \geq 1$, where x_0 and x_1 are given. Assume that $|e_k| \leq |e_{k-1}|$ $\forall k \in \mathbb{N}^+$, and prove for the Weighted Newton Method that

$$|e_{n+1}| \leq \frac{4}{3}|e_n| + \frac{2}{3}|e_{n-1}|$$

(Hint: using Taylor's Theorem and some algebra find an upper bound for $\left|e_n + \frac{f(e_n)}{f'(e_n)}\right|$)