CONFORMAL MAP REFERENCES

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This document is designed as a list of maps that take certain regions to other regions, generally conformal and holomorphic. This is designed predominantly as a reference for the University of Tennessee Analysis Preliminary Examination.

Full Strip

(1) The map e^z takes the strip $\{z \in \mathbb{C}, -\pi < \Im z < \pi\}$ to the set $\mathbb{C} \setminus (-\infty, 0]$

Imaginary Axis

(1) The map z^2 sends to negative real axis

Negative Real Axis

(1) The map $\sqrt{z}$ sends to imaginary axis

The Right Half-Plane

(1) The map $\frac{1+z}{1-z}$ maps onto $\mathbb{D}$.
(2) The map iz maps onto the upper half-plane
(3) The map z^2 maps onto $C \setminus \{z \in \mathbb{R}, z \leq 0\}$

Half-Strip $\{x + yi, x < 0, 0 < y < \pi\}$

(1) The map e^z takes this half-strip to the upper half-disc

The Upper Half-Plane

(1) The map $-iz$ maps onto the right half-plane
(2) The map $\frac{i-z}{i+z}$ maps onto $\mathbb{D}$

$\mathbb{C} \setminus \{z \in \mathbb{R}, z \leq 0\}$

(1) The map $\sqrt{z}$ maps onto the right half-plane

$\mathbb{D}$

(1) The map $\frac{1}{z}$ maps onto the region $\{z \in \mathbb{C},\ |z| > 1\}$
(2) The map $\text{Log}(z)$ with the $(-\pi, \pi)$ branch of the logarithm maps onto $\mathbb{C} \setminus [-\pi i, \pi i]$.
(3) The map $i\left(\frac{1+z}{1-z}\right)$ maps onto the upper half-plane
(4) The map $-i\left(\frac{1+z}{1-z}\right)$ maps onto the lower half-plane

The Upper Half-Disc

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(1) The map $\text{Log}(z)$ with the $(-\pi, \pi)$ branch of the logarithm maps the upper half-disc to the strip $\{x + yi, x < 0, 0 < y < \pi\}$

Quarter-Plane $\{z \in \mathbb{C} : \Re z > 0, \Im z > 0\}$

(1) The map $\sin^{-1}(z)$ maps this quarter-plane onto the half-strip $\{z \in \mathbb{C}, \Im z > 0, 0 < \Re z < 1\}$

General Strategies I've Discovered for Problem-Solving

(1) At each step ask yourself if you want to map portions of the real line onto portions of the real line or to something else, like a circle
(2) If a **clircle** is a line or a circle, then fractional linear transformations map clircles onto clircles
(3) A strip should remind one of using an exponential map to get to $\mathbb{D}$

REFERENCES

[Sar] D. Sarason, “Complex Function Theory," 2nd edition (2007). American Mathematical Society.