Example: You decide to split the numbers $\{1, 2, 3, \ldots, 200\}$ into ~~[scribble]~~ r groups. There is guaranteed to be three different multiples of 8 in one of the groups. What is the largest possible value of ~~[scribble]~~ r?

Solution: $200/8 = 25$, so there are 25 multiples of 8 amongst the groups. We sort all 200 numbers, but the ones that are not multiples of 8 do not matter.

~~What~~ We want $k+1=3$ (in formula), or $k=2$. We know also $n=25$, and want r.

~~So the~~. Inequality is $kr<n$, or $2r<25$. Largest integer satisfying this inequality is $r=12$.

So we can split the numbers into up to 12 groups.

MATH 113 Special Topic ~ Pigeonhole Principle

Intuitive Idea: if you split a [finite] set of items into a [finite] set of groups, one of the groups must have a certain number of items.

Official Statement: if n objects are placed into r boxes (where $r < n$) then at least one of the boxes contains more than one object

More Generally: if n objects are placed into r boxes (where $kr \le n$, $k \in \mathbb{N}^+$) then at least one of the boxes contains at least $k+1$ objects.

Question: What happens if $r > n$? Some boxes may be empty.

Example: An animal shelter takes only cats, dogs, and rabbits. If there are 10 animals in total, there must be at least 4 of one kind.

Here $n = 10$, $r = 3$, $k = 3$.

Can also see reason indirectly: what if there are only 3 of each animal?

Example: Pick any 50 positive integers out of the set $\{1, 2, 3, \ldots, 98\}$. Two of them must sum to 99.

Solution: For any number, there is another in $\{1, \ldots, 98\}$ so the sum is 99.

$73 + 26$ $\quad$ $15 + 84$ and so on.

Make these the groups (pairs). There are 49 of them, and 50 numbers appear. $n = 50$, $r = 49$, so one pair is complete and we are done.