Math 113 Stretch 1

Spring 2023, Section 4, Mr. Joshua Siktar

January 30, 2023

1 Recursive versus Non-recursive [7 points]

a. Circle the formulas below that are **recursive**

$$F_n = F_{n-1} + F_{n-2}$$

$$S_n = n^2$$

$$G_n = 2G_{n-1}$$

$$T_n = \frac{n(n+1)}{2}$$

b. In the above list, put a star next to the formula for the **triangular numbers**.

c. Explain in words the difference between a recursive formula and a non-recursive formula.

2 Infinities [12 points]

a. True or false? The set of numbers $\{1, 2, 3, 4, \ldots, 2023\}$ is finite. If false, use a different term from class to describe the set.

b. True or false? The set of numbers $\{1, 2, 3, 4, \ldots\}$ is finite. If false, use a different term from class to describe the set.

c. Draw an arrow diagram to give a one-to-one correspondence between the natural numbers and the integers.

d. Draw an arrow diagram to give a one-to-one correspondence between the natural numbers and the perfect squares.

e. In one or two sentences, explain what conclusion Cantor's Diagonalization Argument allowed us to make.

3 Cups [10 points]

The tetrahedral numbers are a three-dimensional version of the triangular numbers. This investigation will clue you in to the mathematics behind the pattern used to construct them. Let's call the sequence H_n.

a. Start building! I want to see the sixth tetrahedral number in a stack of cups (the first tetrahedral number is 1), and a list of values for the first six tetrahedral numbers (H_1, H_2, H_3, H_4, H_5, and H_6). When you have reached this point, call me over and I will initial your papers to indicate that you should get points for completing this part of the activity.

b. Explain in words what you are doing to the stack of cups to go from one tetrahedral number to the next.

c. **Bonus [up to 3 additional points]:** A non-recursive formula for the tetrahedral numbers is $H_n = \frac{n(n+1)(n+2)}{6}$. Using this information, try to find a RECURSIVE formula for the tetrahedral numbers. *Hint: what do all recursive formulas have in common?*