Math 113 Stretch 2

Spring 2023, Section 4, Mr. Joshua Siktar

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1 Conjectures Scavenger Hunt [7 points]

Remember that Goldbach's Conjecture claims that every even number greater than 2 can be written as the sum of two prime numbers. Here we'll do some further discussion of this conjecture.

a. Write 4 as the sum of two prime numbers. Is that the only way you can do it?

b. Write 22 as the sum of two prime numbers in three different ways.

c. **Cousin primes** are prime numbers that differ by 4. Subtract them, and you will get 4. List two examples of pairs of cousin primes.

2 LCMs and GCDs [8 points]

Find the LCM and GCD of 60 and 32. Show the Venn Diagram you make to help figure out your answer.

3 Fibonacci Patterns [7 points]

Remember that the Fibonacci sequence starts with $F_1 = 1$, $F_2 = 1$, and has the recursive formula $F_n = F_{n-1} + F_{n-2}$.

a. Write the first twelve Fibonacci numbers, starting from $F_1 = 1$. Feel free to use a calculator for the larger ones.

b. In your list in part a, circle or underline the numbers that are even.

c. Describe the pattern that you see for the numbers you underlined.

d. Think about the recursive formula: why do you think this pattern takes place?

4 Golden Ratio and Telescoping [5 points]

Remember that we talked about the telescoping technique, which showed us that

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \dots}}}} = \phi,$$

with ϕ being the Golden Ratio.

a. Explain in words, the best you can, the steps that were used in telescoping. Feel free to re-copy the above equation and draw on it if it helps you.

b. **Bonus [up to 3 additional points]:** Use the telescoping technique on

$$y = \sqrt{3 + \sqrt{1 + \sqrt{3 + \sqrt{1 + \dots}}}}$$

Once you find a quartic equation involving y, you may stop. Your goal is to find an equation with a y^4 in it.